Providing an Evolutionary Approach for Multi-objective Portfolio Optimization Problem by Using Evolutionary Algorithm Multi-objective NSGA II

Maryam khajouri*, somaye khajouri 1, Mohammad saraei 2,3

1Islamic Azad University of khalij-fars khorramshahr, Iran.
2Young Researchers and Elite Club, Arak Branch, Islamic Azad University, Arak, Iran.
3Department of Computer Engineering, Faculty of Engineering, Islamic Azad University, Arak Branch, Arak, Iran

Abstract

An important decision which companies should make is to create the optimal portfolio. Therefore, researchers have become interested in the selection of a portfolio with a high rate of return and controlled risk. The aim of this study is to use multi-objective meta heuristic algorithms for portfolio selection. NSGA-II, NRGA, and SPEA2 were used to form the proposed portfolio. Then VaR (value at risk) was regarded as a criterion for measuring risk. For this purpose, the information was collected from 20 companies operating in the stock market. They were selected from www.yahoo.com. The results indicated the efficiency and accuracy of the proposed algorithm in comparison with other algorithms.

Keywords: Portfolio, Multi-objective Optimization, Investment, NSGAII Algorithm
Introduction

Investors are constantly looking for ways to earn income from their investment. In recent years, many efforts have been made to lead investors to invest in an appropriate ways and many models offered. Portfolio optimization and diversification have become instruments in the development and understanding of financial markets and financial decision-making. Portfolio is combination of different stock to hold for investment. Selection of portfolio to minimize risk while maximizing returns is one of the main concerns of investors in financial markets. Stock returns varies in different periods and do not have stability process. Investors are risky because of the uncertainty about stock future returns so generally, the risk is measured based on the variance of returns. The exact method for selection the optimal portfolio is inefficient and therefore meta-heuristic algorithms considered for solving this problem.

One of the most important ways to reduce portfolio risk is diversified investment and continuous monitoring of the status of stocks and other securities included in the portfolio. It means, investors always check the factors affecting the stock price and other types of securities and analysis whether a company shares in the portfolio have still the possibility of future growth or should sell the shares and purchase another company stock that has better growth opportunities and profitability.

Portfolio is the right combination of stocks or other assets that an investor has bought them. The pecuniary value of the portfolio of any natural or legal person is portfolio value. The most important factor for pricing investment companies listed on the Stock Exchange is the company portfolio value. The purpose of investors for portfolio is tolerance and gets the most returns at least risk. Optimal portfolio selection (OPS) is one of the most important issues in financial science and investments and has many applications in financial planning and decision-making. Harry Markowitz theory of portfolio selection was the main and most important success in this regard. Since Markowitz presented his version of the model, a lot of changes and improvements created in the view of people for investment and portfolio and used as an efficient tool for portfolio optimization. Markowitz suggested that investors consider both risk, return, and choose the capital allocation between the various investments opportunities based on the interaction between these two factors. However, Markowitz portfolio theory provides only solution for capital allocation. In the capital markets, there are hundreds of different types of capital quality.
from very good to very bad and investors facing information influx, so it is difficult to choose the best. Markowitz model solved by using mathematical programming models but when real-world constraints such as the large number of assets, restrictions on stock etc. added to the model, search space become large and discontinuous and it is virtually impossible using mathematical models.

Regarding the number of objective functions and optimization criteria, optimization problems are divided into two groups, the first of which includes single-objective optimization problems. The second group includes multi-objective optimization problems. In the first group, the goal is to solve the improvement problem of a single performance index, the minimum and maximum values of which reflect the quality of solutions completely. However, one index cannot sometimes be relied upon to score an assumed solution to an optimization problem. In such cases, we have to define several objective functions or performance indices and optimize them at the same time. Multi-objective optimization is one of the most widely-used research techniques in optimization problems. The goal of the current problem is to maximize the rate of return and minimize risk. Many methods have been proposed to solve such problems. They can generally be classified as classic and evolutionary methods, the first of which often decreases a multi-objective problem to a single-objective one. The second one solves a multi-objective optimization problem in a really multi-objective way.

Multi-objective models are used in a continuous set of solutions for design. Finally, they result in a mathematical model which can provide planners with design options. In this case, a problem is modeled and solved mathematically. Given the fact that several objectives should be optimized at the same time in such models, a goal programing method or a composite programing method can be employed to convert a multi-objective problem into a single-objective one. Then the problem is solved using available techniques. Given the fact that the mathematical model grows exponentially by increase the number of indices and land dimensions in such decision-making problems, it appears that there will be no definite methods to solve the problem accurately within a reasonable period. In such problems known as nondeterministic polynomial time problems, artificial intelligence methods and meta heuristic algorithms are generally used to find a solution. These processes do not guarantee an optimal solution; however, they are the only available methods to solve such problems. Therefore, meta heuristic methods such as the genetic algorithm and the ant colony optimization are of special importance.
Related Work

Foundations of portfolio optimization theory and modern portfolio theory (MPT) based in the early 1950s and by Harry Markowitz and many modern achievements of this branch of financial science indebted to his studies. To solve the problem of portfolio optimization, various tools and algorithms proposed and can use and also includes classical optimization algorithms [1-3] as well as the smart optimization algorithms (meta-heuristic). Stock portfolio problem in recent decades has been favorable issue for many researchers in industrial engineering [4-5], computer [6] financial [7-9], operations research and almost solved as a classic problem with met heuristic algorithms such as genetic [10-12], particle swarm [13,14], colonies of bees [15], ant colony [16,17] and Memetic [18]. Haghighi and Kazemi [19] solving problem using the ant colony algorithm and variance model to determine the type of error. In addition, Chang and colleagues [20] presented meta-heuristic methods for solving optimization of their portfolio in which genetic algorithms used different stock portfolios that their risk calculated in different ways also Wang et al [21] using particle swarm algorithm for optimization. The objective function was multi-functions and constraints was nonlinear, and the answer compared with genetic algorithm and particle swarm algorithm had a better answer. In addition, the same problem was solved in [22] using an applied approach to TOPSIS.

Problem Design
Portfolio optimization is to choose a combination of financial assets so that risk is minimized and caused the maximum return on the investment. Components of risk and return of capital are two important issues in the optimal portfolio for the investment choice. Selection of the optimal capital often done by exchange between risk and return and if the risk of capital is further, investors will expect higher returns.

Problems arising from the portfolio

Various factors involved in the formation of profitable portfolio. Investors to avoid risk in their profits mainly suggest that use several shares instead of a single share in portfolio because in this case, we can reduce the risk and cover loss of a share by profit of another share. We should consider many factors to choose portfolio but there are other important factors: The aim of investing and investment condition. This means that the investors prior to the formation of each portfolio must determine the degree of risk appetite. This degree can change stock fate. Meanwhile, purpose of investors and amount of invested capital are also very important.
It can said that in order to determination of efficient portfolio addition to variables affecting the stock, investor's condition also looked.

**Risks arises portfolio risk**

Stock selection is important for investors to buy and hold it and they are interested to know what the greatest danger is do value face? Therefore, the important purpose of this study is the most important issue that investors facing: what is the size and degree of risk and does it have constraint? (VaR) \(^1\) is a measure criterion of risk that estimates expected loss on specific investment position and certain confidence level. Value at risk by considering the restrictions can be useful to reduce losses.

**The aim of minimizing the risks in the portfolio**

We expect Minimum risk and maximum profit in investing. Investors, whether natural or legal, to reduce the risk of investments of other companies buy varied shares (investment portfolio). Stock portfolio adjusts effects of variations on stock returns and risk. Investors are always looking for risk measurement to obtain optimal investment decisions. Therefore, risk minimization is particular important issue.

**NSGA II Algorithm**

**The Main Concept of Algorithm**

The multi-objective non dominated sorting genetic algorithm is a well-known and widely-used algorithm in multi-objective optimization. After the first version of this algorithm was introduced in 1995, the presenters (out of whom Deb was the most famous) proposed the second version named NSGA-II in 2002. In addition to all functions of NSGA-II as a meta heuristic algorithm, it can be regarded as a model by which many other multi-objective optimization algorithms are formed. NSGA-II has been used many times to create newer multi-objective optimization algorithms due to its unique way of dealing with such problems. Undoubtedly, this algorithm is one of the most important methods of evolutionary multi-objective optimization techniques which can be named the second generation of such approaches. This algorithm is mainly characterized by the following features:

\(^1\) Value at Risk
• Defining a cumulative distance as an alternative feature for methods such as fitness sharing
• Using the binary tournament selection operator
• Saving and archiving non dominated solutions obtained in the previous steps of algorithm (elitism)

The components of NSGA-II are as follows:

1. Creating the initial population and sorting it according to domination conditions
2. Calculating the fitness criteria
3. Calculating the swarm distance
4. Performing crossover and mutation to produce new children
5. Combining the initial population and the population obtained from crossover and mutation
6. Replacing the population of parents with the best members of the population combined in the previous step

First, lower-order members replace previous parents, and then they are sorted according to the swarm distance. The initial population and the resultant population are ranked. Then a lower-order part of population is deleted. In the next step, the remaining population is sorted according to the swarm distance. Here sorting is performed inside a façade.

7. All of these steps are iterated until the desirable generation (or optimal conditions) will be obtained.
Algorithm 1. Pseudo-code of NSGA II Algorithm

The proposed algorithm

Basic instructions on the financial investment are a variety selection of different items to flood of capital as investors invest in different types of capital. Portfolio offers diversified exposure to minimize risks while maximizing efficiency. Many methods can use to solve portfolio optimization. One of these methods is change a multi-objective optimization problem into a single-objective optimization problem. This method can divide into two sub-types: In the first method, one of the most important criteria selected as the objective function to optimize while the rest of the functions defined as narrowly conditions. Alternatively, only an evaluation function created to multiple criteria. [23]. The first method is defined by Markowitz and called standard mean-variance model [24]. In this model, the size of the portfolio risk defined as a purpose function and the average return on capital consider as constraint that it can formulate as follows [25]:

\[
\min \sigma_{R_p}^2 = \sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{Cov}(\bar{R}_i, \bar{R}_j) \quad (1)
\]
Subject to

\[ \bar{R}_p = E(R_p) = \sum_{i=1}^{N} w_i \bar{R}_i \geq R \quad (2) \]

\[ \sum_{i=1}^{N} w_i = 1 \quad (3) \]

\[ w_i \geq 0, \forall i \in (1,2,\ldots,N) \quad (4) \]

Where \( N \) is the number of capitals and \( \bar{R}_i \) is average return on assets \((i)\) and \( \text{Cov} (\bar{R}_i \bar{R}_j) \), respectively are the covariance of return on assets \(i\) and \(j\). \( w_i \) weight control variable is the ratio of capital invested in \(i\). Capital constraints guaranteed by equation (2). In this model, the goal of minimizing the risk of portfolio \( \sigma_p^2 \) is set for a return \( R_p \). Here we use the first method, described above, and the second explanation is beyond the scope of this article. The solution structure includes random numbers in an interval generated to create initial solutions to the algorithm. In the proposed method, local search and movement were employed to improve the results and extend the range of discoveries in the proximity of each solution generated by NSGA-II. In this algorithm, the initial population is created randomly first. Then it is sorted according to domination rules, and the swarm distance is calculated. A mean of distance from the two solutions located on the sides of a solution to each function is calculated to estimate the densities of existing solutions near a specific one. The swarm distance is the numerical values obtained by calculating the square space (double-objective problems) in the proximity of a solution \((j)\) and using the closest neighbors. It can be calculated through the following formula.

After applying mutation and crossover on the population of each generation, the initial population is merged into the resultant population. Finally, the parent population is exchanged with the best members of the merged population. Then the problem is solved like a multi-objective problem.

\[ D_{j}=\sum_{i=1}^{n} \frac{f_i(j-1)-f_i(j+1)}{f_{i\text{max}}-f_{i\text{min}}} \quad (5) \]
Swapping mutation

In this method, the two random elements selected and then we move them together. This mutation is one of the most common mutations. Figure (2) show a permutation changes desired by applying a mutation.

Figure 1. Swapping Mutation Sample

![Figure 1. Swapping Mutation Sample](image1)

Figure 2. Flowchart of Proposed Algorithm

![Figure 2. Flowchart of Proposed Algorithm](image2)
Simulation

In this article, improved NSGA-II algorithm to solve the problem of portfolio optimization executed using simulator platform MATLAB R2015 and Intel CORE i5, 4GB memory RAM, Windows 7valid for 20 companies from global stock market that provided by Yahoo and for standard data with an average of 15 times. In this algorithm we used The combination roulette wheel selection base on rating of wheel . In this algorithm we created Pool mating of the parent population with the Choose the best solution random, Then we compared this problem with famous algorithm such as NRGA, -SPEAⅡ. The required parameters for the proposed algorithm and other algorithms are visible in the table (1-3) and The results of the comparisons provided in Table (4). The lowest risk and the highest efficiency Related to proposed algorithm in addition the risk and efficiency Diagram of the algorithms are shown in Figure 3-5.

Table (1). Parameters used in SPEA2 algorithm

<table>
<thead>
<tr>
<th>PARAMETERS</th>
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<tr>
<td>MAXIMUM NUMBER OF ITERATIONS</td>
<td>400</td>
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<td>POPULATION SIZE</td>
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<td>CROSSOVER PERCENTAGE</td>
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Table (2). Parameters used in NRGA algorithm

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Table (3). Parameters used in NSGA-II algorithm

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Conclusion

This paper used NSGA-II, SPEA2, and NRGA to solve the problem of multi-objective portfolio in order to decrease risk and increase the rate of return. Then the performances of these three algorithms were compared. NSGA-II is better at searching into the problem space. It is also completely easy for implementation. A new mutation-based version of NSGA-II named SWAP was introduced in this study to solve the problem of portfolio by using the local search.
According to the results, the proposed algorithm decreased risk and increased the rate of return in comparison with other standard algorithms. Table (4) compares the three algorithms in risk and the rate of return. Min, Max, STD, mean and response time were compared regarding risk and the rate of return. The results show that the proposed method is more optimal than other algorithms. Moreover, the proposed algorithm has greatly improved by increasing the number of iterations.

Table (4). Results of return and risk of the problem compared with other algorithms

<table>
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<tr>
<th>Factor</th>
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<th>SPEA2</th>
<th>NRGA</th>
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<tr>
<td></td>
<td>Min</td>
<td>0.011258</td>
<td>0.015689</td>
<td>0.0097779</td>
</tr>
<tr>
<td>Risk</td>
<td>Max</td>
<td>0.019217</td>
<td>0.022592</td>
<td>0.01992</td>
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<td></td>
<td>S.T.D</td>
<td>0.0023944</td>
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<td></td>
<td>Mean</td>
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<td>0.01851</td>
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</tr>
<tr>
<td></td>
<td>Time</td>
<td>210.563165</td>
<td>170.351281</td>
<td>145.934843</td>
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<tr>
<td>Return</td>
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<td>0.0033278</td>
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<tr>
<td></td>
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<td>Time</td>
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</table>
Figure 3: Risk and Efficiency Chart of NRGA Algorithm

Figure 4: Risk and Efficiency Chart of SPLA2 Algorithm

Figure 5: Risk and Efficiency Chart of NSGA-II Algorithm
Suggestions

Other pseudo-biological algorithms such as bat, weed optimization (iwo), shrimp swarm algorithms can be used to solve this problem by dividing the it into single-objective problems. Then the results can be compared with other algorithms. In addition, the proposed algorithm can be combined with high-speed point algorithms such as Tabu search, the greedy algorithm, the neighborhood, multiple or triple mutation operators, and other techniques in order to obtain better results. Although it appears that combining the proposed algorithm with other metaheuristic algorithms such as Tabu search, ant colony optimization, and the bee algorithm will produce better results than multiple or triple improvers. On the other hand, this algorithm can be used to solve other optimization problems such as the value of conditional risk and portfolio in stock exchange securities. Future papers are expected to investigate other ideas and make them practical.

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References


**AUTHOR**

**M. Khajouri** was born in abadan, Iran, in 1979. she received the B.S degree in 2013 and Student Financial in the M.Sc degree in Islamic Azad University of Khorramshahr-khalij fars Branch, Khorramshahr, Iran, Of the Year 2016. her research interests include: Portfolio, Securities Exchange, and Securities Exchange Technology Management. She has more than 3 international journals and Conference papers in the Financial Scope, Shi is a member of iranian accounting association.